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Flux qubit with a quantum point contact

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Abstract

Andreev bound states in superconducting quantum point contacts (QPC) can be accessed for manipulation and measurement by embedding the QPC in a superconducting loop. We discuss the characteristics of such a device suitable for qubit operation, methods of manipulations with the Andreev levels, and qubit coupling. © 2001 Elsevier Science B.V. All rights reserved.

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1. Introduction

Implementation of Josephson junctions for quantum computation is an interesting theoretical and a challenging experimental problem. Two basic realizations for a superconducting qubit have been proposed based on the operation either with flux or charge (for review, see Ref. [1]). The flux qubit is, in the simplest case, a hysteretic rf SQUID containing one Josephson junction with small charging energy E_C compared to the Josephson energy E_J, $E_C \ll E_J$ [2,3]. The qubit is biased at half-integer external flux, $\Phi_e = (n + 1/2)\Phi_0$, and operates with two degenerate flux states corresponding to clockwise and counter-clockwise circulating currents. The tunnel coupling between these flux states is provided by charge fluctuations on the junction capacitance (macroscopic quantum coherence (MQC)) [4]. Read-out of the qubit state is realized by measuring the induced flux by means of a SQUID magnetometer. Various modifications of the flux qubit have been proposed, e.g., using SQUIDs with three junctions [5] or with d-wave superconducting junctions [6,7]. Charge qubits operate with charge states on a superconducting island in the opposite regime, $E_C \gg E_J$ [1]. The quantum mechanical coupling of the charge states is induced by the Josephson tunneling into a bulk superconducting electrode. The state of the qubit is measured by detecting the charge on the island.

In both the realizations of a superconducting qubit, the quantum fluctuations of charge or phase play the central role while the Josephson junction is assumed to be a macroscopic object, which does not exhibit any quantum fluctuations. In this paper we discuss a third possible realization of a superconducting qubit, the Andreev level qubit (ALQ), which is based on the quantum fluctuations of

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current in a quantum point contact (QPC) embedded in a superconducting loop [8].

It is theoretically well established [9–11] and convincingly confirmed by experiments on atomic-size contacts [12–14] that the Josephson current through superconducting QPCs is primarily carried by Andreev bound states. These states are formed in the contact due to Andreev reflections by the large gradient of the superconducting phase across the contact. In single-mode QPC, the amount of Andreev bound states is limited to a single pair spanning the Hilbert space of the qubit. In a perfectly transparent QPC (reflectivity $R = 0$), the Andreev bound states have energies $E(\phi) = \pm\Delta\cos\phi/2$, are eigenstates of the current operator, and carry well defined currents,

$$I = \frac{2e}{\hbar}\frac{dE}{d\phi}, \tag{1}$$

which flow in opposite directions. When the QPC is embedded in a superconducting loop forming an rf SQUID, these states generate two independent flux states, $|\uparrow\rangle$ and $|\downarrow\rangle$, see Fig. 1. In QPCs with finite reflectivity, $R = 1 - D \neq 0$, these current states undergo hybridization, which leads to the opening of a gap in the Andreev level spectrum, $E(\phi) = \pm E_a$, $E_a = \Delta(1 - D\sin^2(\phi/2))^{1/2}$, and also to strong quantum fluctuations of the current. In this case, Eq. (1) merely gives an expectation value for the current, $\langle I\rangle = (2e/\hbar)(dE/d\phi)$. Consequently, the induced flux in the SQUID exhibits quantum fluctuations, and the state of the qubit appears as a coherent superposition of the two flux states $\alpha|\uparrow\rangle + \beta|\downarrow\rangle$ ($\alpha = \pm\beta$ for $\phi = \pi$).

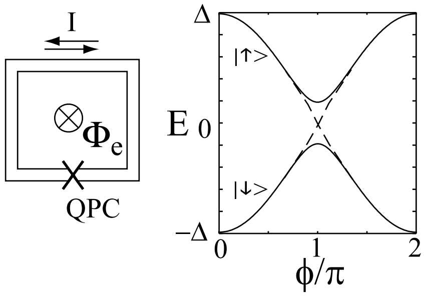

Fig. 1. Left: Sketch of the ALQ—a non-hysteretic rf SQUID with a QPC. Right: The energy spectrum of the QPC with finite reflectivity ($R = 0.04$) (—), appears as a hybridization of the current states $|\uparrow\rangle$ and $|\downarrow\rangle$ ($R = 0$) (- - -).

The SQUID geometry allows manipulation and measurement of the state of the ALQ. By ramping the external flux or by applying properly designed rf flux pulses, one can drive Andreev levels from the ground state to an arbitrary excited state, and then this state can be measured by monitoring the induced flux, similar to the MQC-based flux qubits.

We emphasize that an appreciable level of the quantum fluctuations, which is the basic idea behind the ALQ, is the consequence of the small number of electronic conducting modes in the QPC: in Josephson junctions with a macroscopically large number of conducting modes these fluctuations are significantly suppressed.

2. The Andreev level qubit

The ALQ is the Andreev two-level system in a QPC. The ALQ is made accessible for manipulations and read-out due to embedding the QPC in a superconducting loop forming an rf SQUID. For such a device, the phase dependence of the average current has been measured by Koops et al. [12] using the controllable-break-junction technique. For qubit operation, the junction and SQUID characteristics must be carefully designed. The most important requirement is small reflectivity of the QPC, $R \ll 1$. In such contacts, biased at phase differences close to π, the Andreev levels lie deep within the superconducting gap, $E_a \sim \Delta\sqrt{R} \ll \Delta$, which makes them well decoupled from the continuum states. In this case of small R, the critical current approaches the upper bound for the supercurrent per conducting mode, $I_c \approx e\Delta/2\hbar$ (it is approximately equal to 0.37 μA for Nb), and the Josephson energy is close to the gap value, $E_J \approx \Delta$. In experimental atomic-size QPCs, reflectivities as small as $R \sim 0.01$ have been achieved [15].

In order to monitor the dynamic evolution of the ALQ by observing the flux, it is convenient to separate the characteristic time scales of the Andreev level dynamics and the dynamics of the loop. To this end, we suppose that the plasma frequency of the SQUID is large compared to the interlevel distance, $\hbar\omega_p \gg 2E_a$. In this adiabatic regime, the induced flux follows the evolution of the ALQ, and the read-out is particularly simple. It is also convenient to choose a non-hysteretic regime for the SQUID, assuming the magnetic energy of the loop, $E_L = (\Phi_0/2\pi)^2 L$, to be large compared to the Josephson energy, $E_L \gg E_J \sim \Delta$. The plasma frequency of the SQUID is in this regime given by the relation $\hbar\omega_p = \sqrt{8E_L E_C} = \hbar c/\sqrt{LC}$. Finally, we assume that the temperature is zero and the plasma frequency is small, $\hbar\omega_p < 2\Delta$, to avoid dissipation due to creation of real excitations. Combining together all the assumptions made, we get the following chain of inequalities,

$$\sqrt{R}\Delta \ll \sqrt{E_L E_C} < 2\Delta \ll E_L. \quad (2)$$

These conditions are equivalent to the requirements for the loop inductivity $L \ll 10^4 \xi_0$ and the junction capacitance that $LC \gg 10^4 \xi_0^2$, where $\xi_0 = \hbar v_F/\Delta$ is the superconducting coherence length of the loop material.

To quickly put ALQ into the ground state it is sufficient to bias the qubit at zero phase difference, $\phi = 0$, because at this point the Andreev levels approach the continuum spectrum. One way to prepare an excited state of the qubit would be adiabatically to switch on the phase difference and to apply, in the vicinity of the working point $\phi \approx \pi$, a small rf-signal with frequency close to the level separation, $2E_a(\phi)$. This will produce Rabi oscillations (nutation) of the Andreev two-level system with frequency

$$\Omega = 2A\sqrt{R}D(\Delta^2/E_a)\sin^2(\phi/2) \approx 2A\Delta,$$

where A is an amplitude of the rf-signal [11]. By applying rf-pulses with controlled duration, one can put the qubit into any coherent excited state. An alternative way to produce excited states is to slowly drive the phase, in the presence of an rf-signal with fixed frequency ω, through the resonant region $2E_a(\phi(t)) \approx \omega$ [16]. After the rf-signal is switched off, the qubit will keep precessing with frequency $2E_a$, until relaxation and dephasing make it relax back to the ground state. The average current for this free precession of the qubit has the dc and ac components $\langle I_{dc}\rangle = |I|(|a|^2 - |b|^2)$ and $\langle I_{ac}\rangle = 8I_Q D\sqrt{R}\sin^2(\phi/2)(\Delta/E_a)\mathscr{R}(a^* b e^{2iE_a t})$ respectively, where a and b are the amplitudes of

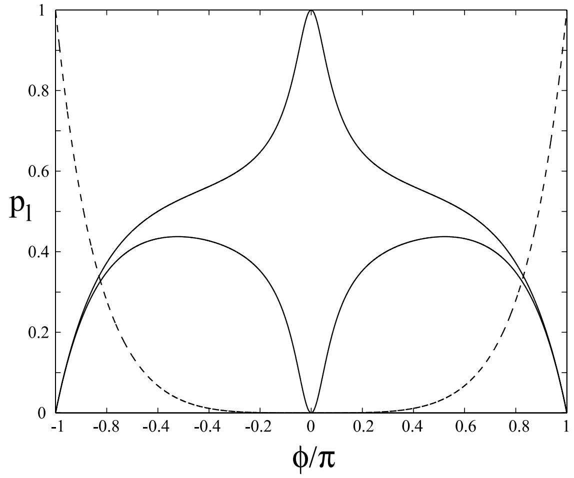

Fig. 2. The probability to find the ALQ in the ground state and the excited state after instant switching of the phase difference (—). The qubit was initially in the ground state at $\phi = \pi$ ($R = 0.01$). The leakage to the continuum, caused by the instant switching given by Eq. (3), is shown with a dashed line.

the precessing state in the Andreev level basis, and $I_Q = e\Delta/2\hbar$.

The relaxation and dephasing mechanisms for the Andreev qubit are similar to the ones for the MQC-based flux qubits (fluctuation of the bias, electromagnetic emission) [17,18]. However, there is an additional relaxation mechanism which is specific for the ALQ involving microscopic interactions with the quasiparticles in the electrodes, primarily with phonons. The rate of electron–phonon relaxation of Andreev levels has been calculated in Ref. [19]. For our case, it is estimated as $\nu_{ph} \sim R^{3/2}\Delta^3/\theta_D^2$, which is smaller than the qubit precession frequency by a factor of 10^4 for $R \sim 0.01$.

The qubit can also be put into a coherent excited state by ramping the external flux. The Andreev level populations after an instant ramping from $\phi = \pi$ is illustrated in Fig. 2 [1] for a qubit which was initially in the ground state. Such ramping will also induce transitions to continuum states, the probability of which is shown by the dashed line in Fig. 2. In the general case of

[1] Fig. 2 has been produced in collaboration with Andreas Käck.

switching between two arbitrary phases, the leakage probability is given by the equation,

$$P(\phi, \phi') = \left(\frac{|\sin(\phi/2)| - |\sin(\phi'/2)|}{|\sin(\phi/2)| + |\sin(\phi'/2)|} \right)^2. \qquad (3)$$

This equation establishes the upper limit for the leakage to the continuum, universal for junctions with different reflectivity. However, if the ramping time is finite, although small compared to the inverse interlevel distance, $\hbar/2E_a$, and large compared to inverse distance to the continuum, $\hbar/(\Delta + E_a)$, then the leakage to the continuum will be exponentially suppressed, while the level populations will be approximately the same as in the case of the instant ramping. Such a ramping regime is possible for junctions with small reflectivity.

3. Coupling of Andreev level qubits

Similarly to the MQC-based flux qubits, ALQs may interact via inductive coupling [20]. However, implementation of multi-mode QPCs gives additional interesting possibilities for qubit coupling due to direct intermode quasiparticle scattering. For instance, a SQUID with a two-mode QPC provides a realization of a two-qubit configuration. The drawback of this realization is the difficulty of controlling the qubit coupling; also independent control of the qubits is not possible. These difficulties can be avoided by considering the multi-terminal junction configuration [21] schematically shown in Fig. 3. Such junctions, which can in principle be fabricated on the basis of gated SNS (S-2DEG-S) structures, will contain two pairs of Andreev levels controlled by two independent external fluxes, Φ_1 and Φ_2. The Andreev level spectrum for weakly coupled identical qubits ($D_1 = D_2$) is given by the equation,

$$E^2 = E_+^2 \pm \sqrt{E_-^4 + TD\Delta^4\left[(\theta_1 - \theta_2)^2 + 4\theta_1\theta_2 \sin^2(\phi_3/2)\right]},$$

where $T \ll 1$ is the interqubit transition probability, and ϕ_3 is the phase difference associated with the flux Φ_3 through the third loop, see Fig. 3; $\theta_{1,2} = (\pi - \phi_{1,2})/2$, $E_\pm^2 = (E_a^2(\theta_1) \pm E_a^2(\theta_2))/2$.

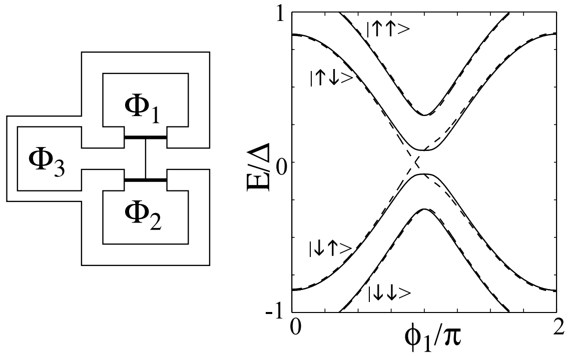

Fig. 3. Left: Coupling of ALQs by means of a four terminal quantum contact. The flux Φ_3 through the third superconducting loop can be used to control the coupling. Right: The two particle spectrum of two interacting ALQs, $D_1 = D_2 = 0.97$, $T = 0.01$ and $\phi_2 = 0.93\pi$. The solid line corresponds to coupled qubits ($\phi_3 = \pi/2$) and the dashed line corresponds to uncoupled qubits ($\phi_3 = 0$).

Using this setup it is possible to control the coupling of the qubits by means of the flux variation through the third loop, and to switch off the coupling by setting $\phi_1 = \phi_2$, and $\phi_3 = 0$.

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